

Progression in Calculations

EYFS – Year 6

September 2023

Introduction

This policy supports the use of the White Rose Maths scheme, Maths Mastery, Sarah Martin and Tara Loughran resources. Progression within each area of calculation is in line with the mathematics guidance: key stages 1 and 2, non-statutory guidance for the national curriculum in England June 2020. This calculation policy is used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.

• Concrete representation— a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.

• Pictorial representation – a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.

• Abstract representation—a pupil is now capable of representing problems by using mathematical notation, for example 12 x 2 = 24.

It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations

The Mastery of Maths

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that are taught and used in EYFS to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

Our Calculation Policy – The Research

- Teachers should have a clear understanding of how children learn maths. Staff need to know the individual children well and develop a culture that supports children's curiosity/thinking and problem solving. (EEF)
- Manipulatives need to be used purposefully and appropriately to have an impact on learning. (EEF)
- All adults in class should have a clear, strong understanding of why a particular CPA model is being taught. (EEF)
- Representations across year groups should be consistent to connect prior learning to new learning. (DFE Maths Recover

The rights of the mathematician

- 1. The right to enjoy mathematics
- 2. The right to have interests and preferences
- 3. The right to make jottings, drawings and working out
- 4. The right to use our own methods and approaches
- 5. The right to use manipulatives and resources
- 6. The right to reason, to talk about maths and be listened to
- 7. The right to make mistakes
- 8. The right to estimate, to guess and to conjecture
- 9. The right to ponder and take time
- 10. The right to be playful

Catherine Gripton

https://blogs.nottingham.ac.uk/primaryeducationnetwork/2020/02/14/the-rights-of-the-mathematician/

Some of the strate	egies in the document below may be used in more	e than one of the CPA representations dependent upon the	context in which they are taught.			
Addition						
		Addition – EYFS				
ELG Number: Children co objects, they add	unt reliably with numbers from 1 to 20, place ther and subtract two single-digit numbers and count	n in order and say which number is one more or one less th on or back to find the answer. They solve problems, includi	an a given number. Using quantities and ng doubling, halving and sharing.			
		EYFS Vocabulary.				
		Number				
Zero, number, one, two	, three to twenty and beyond, teens numbers, e	eleven, twelve twenty, none, how many? count, count	up) to, count on (from, to), count back			
	(from, to), count in ones, twos, fives,	, tens, is the same as, more, less, odd, even, few, pattern, pa	air			
		Addition				
Key language which shoul	d be used: total, parts and wholes, plus, add, alto	gether, more than, equals, 'is equal to' 'is the same as' colu	nn, place value, counting forward, more,			
	and, make, total, double	e, most, count on, number line, part, part, whole				
Objectives	Concrete	Pictorial	Abstract			
• Find the total		Two groups of pictures so children				
number of	Use toys and general classroom resources for	Can count the total.	A focus on symbols and numbers to			
items in two	children to physically manipulate,		form a calculation.			
groups by						
counting all of	State 🗱 🦛		5+2=7			
them. (including			312 /			
doubling.)	group/regroup	2 ₂ 2 2 ₂ 2				
	8. out/1.08. out/1					
			3			
			part			
	Use specific math's resources such as		2			
	counters, cubes, rekenrek etc.		whole 2			
			part			



	arms for example 5 (show 5 fingers) + (make a + with arms) 2 (show 2 fingers) = (make that with arms so both arms straight across our bodies) 7 (show 7 fingers)							
		Addition Year 1						
add, more, plus, and, ma	Year 1 Vocabulary add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part, whole, parts and wholes, plus, add, altogether, more than. 'is equal to' 'is the same as' column. place value, counting forwards							
Objectives	Concrete	Pictorial	Abstract					
Combining two parts to make a whole: part- whole model	Use cubes to add two numbers together as a group or in a bar. (Some children may still need to use real objects) Use part-part-whole model.	Image: Second system Image: Second system <td< td=""><td>5 part 5 part 5 part 4 + 3 = 7 shown above to move into the abstract. 10 = 6 + 4</td></td<>	5 part 5 part 5 part 4 + 3 = 7 shown above to move into the abstract. 10 = 6 + 4					









Addition Year 2											
Vocabulary											
add, more, p	add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens										
Objectives	Concrete	Pictorial	Abstract								
Adding 3 1-digit numbers	<pre>4+7+6=17 Put 4 and 6 together to make 10. Add on 7. Following on from making 10, making 10 with 2 of the digits (if possible) then add on the third digit.</pre>	Add together three groups of objects. Draw a picture to recombine the groups to make 10.	4 + 7 + 6 = 10 + 7 $= 17$ Combine the two numbers that make 10 and then add on the remainder.								
	When adding 3 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity.	7 + 6 + 3 = 16 $7 + 6 + 3 = 16$ 16	7 6 3								
Adding a 2-digit number and ones	17+5=22 Use ten-frame to make magic ten.	Use the part-part-whole and number line to model.	17 + 5 = 22 17+5=22 17 5								

	16 + 3=	tens ones 16 + 7 +4 +3 16 20 23	
	Children explore the pattern 17+5=22 27+5=32		
		24 + 4 = 28 Bar model.	
Adding a 2-digit number and multiples of 10	25 + 10 = 35 Explore that the ones digit does not change	Dienes may be used above the number line initially. The calculation will be shown alongside the number line to see the connection. 27 + 20= 47	27 + 10 = 37 27 + 20 = 47 27 + = 57
Adding two 2-digit numbers within 100	25 + 26 = 51	25 + 26 = 51	25 + 26 = 51 20 + 20 = 40 5 + 6 = 11 40 + 11 = 51 Leading on to adjusting to make a

			multiple of 10. 25 + 26 = 51 30 + 21 = 51					
		Addition Year 3						
	- This work revises and reinforces ideas from Key Stage 1, including the focus on place value							
Vocabulary addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens, hundreds, decimal, decimal point, plus, one more, two more ten more one hundred more, how many more to make? How many more is than? How much more is?								
Objectives	Concrete	Pictorial	Abstract					
Add numbers with up to 3-digits, using formal written methods of columnar addition Column addition (no regrouping)	269 + 500 = 769 Using manipulatives (dienes, counters) children to line up hundreds, tens and ones. Move to using place value counters	Children move to drawing dienes. $\begin{array}{c c} & t & 0 \\ & t & t & 0 \\ & t & t & 0 \\ & t & t & t \\ & t & t$	Add the ones first, then the tens, then the hundreds. 2 2 3 + 1 1 4 3 3 7					



			536 $+ 85$ 621 11
		Addition Year 4	
add addition more nlu	is increase sum total altogether double near d	Vocabulary ouble near double half halve tens hundreds decimal deci	mal point how many more to make 2
Objectives	Concrete	Pictorial	Abstract
Add numbers with up to 4 digits Using formal written methods of columnar addition where appropriate add numbers with up to 4 digits (with exchange)	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. Thousands Hundreds Tens Ones I housands Hundreds I housands Thousands Hundreds I housands I housands I housands I	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.	1,378 + 2,148 = 3,526 Continue from previous work to carry hundreds as well as tens. 1 3 7 8 + 2 1 4 8 3 5 2 6 1 1 1 1

	7212+4592= + 4 5 9 2		
Add decimals with 2 decimal places, including money. When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20	Calculating with decimal numbers Assign different values to dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these. f = 1 $f = 0.1$ $e = 0.01Place value counters are another usefulmanipulative for representing decimalnumbers.24.2 + 13.4 =$	Draw representations using place value grid.	As the children move on, introduce decimals with the same number of decimal places and different. 401.2 + 26.85 + 0.71 = $4 0 1 . 2 0$ $2 6 . 8 5$ $+ 0 . 7 1$ $4 2 8 . 7 6$ 1 Money can be used here. $\boxed{\underbrace{1}}$ $\boxed{\underbrace{1}}$



		Α	ddit	ion Yea	ar 5						
	Vocabulary										
add, addition, more, plus, increase, sum, total, altogether, score double, near double, how many more to make? Equals, sign, is the same as, tens, hundreds, thousands,											
	millions, ones, tenths, inverse										
Objectives	Concrete					Pictor	ial			Abstract	
Y5—add numbers with	As year 4. Introduce/recap decimal place				1						
more than 4 digits.	value counters and model exchange for		SU	red	spug	spu	eds	2	s	104,328 + 61,731 =	166,059
	addition.			Innd	Tel	onso	undr	Ter	ŏ		
Add decimals with 2	tens and the hundradthe			+=	÷	÷	Ĩ				
decimal places,	tents ones tenths indicating			O		300	00	00	00		
including					000	0	600	~ 0	60	8105	9
money.					000	Ŭ	000	00	00	344	9
		L.	24			1	0	Ū	0	5,66	0
In Year 5, pupils are	For this method start with the digit of least									15,30	
expected to be able to	value because if regrouping happens it will									+20,55	
use formal written	affect the digits of greater value.									120.57	9
methods to add whole						?				1111	
numbers with more	Les 1000 Pacanto Logen Ter Cris Image: State of the state of	-				i			_		
than four digits as well	3 4 6 2 3 💁 🧠 🖉 🙍 📈			10	4,328			61,731			
as working with	+ 5541	_									
numbers with up to											
three decimal places.		For some skilderer have some signer of adding should be the site									
Pupils should think			ietu	of doci		kpenen coc Thi			ing this in		
about whether this is	3 4 6 2 3	a var	iety onto	of ueci	illai pia	ces. III		tes pull			
the most efficient	+ 5 5 4 1 <u>1 1</u> $ -$	C	onte	ext whe	in addir	ig mone	ey and c	uner m	easures.		
method, considering if											
mental methods											
would be more											
effective.	Compline the counters in each column and										

regroup as needed: Decimal numbers: $3 \ 4 \ 2 \ 5 \ 1 \ 5 \ 4 \ 4 \ 5 \ 6 \ 0 \ 1 \ 2 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6 \ 6$	Children should have abstract supported by a pictorial or concrete if still needed. Insert zeros for place holders when $\begin{array}{r} \hline 2 & 3 & 6 & 1 \\ \hline 9 & 0 & 8 & 0 \\ \hline 5 & 9 & 7 & 7 & 0 \\ \hline 4 & 1 & 3 & 0 & 0 \\ \hline 9 & 3 & 5 & 1 & 1 \\ \hline 2 & 1 & 2 & 1 \\ \hline \end{array}$ using decimals.

Addition Year 6									
	Vocabulary								
add, addition, more, plus	, increase, sum, total, altogether, score double, n	ear doub	le, how	many n	nore to	make	? Equa	als sign =, is t	ne same as, tens, hundreds, ones, tenths,
	hun	dredth, t	housan	idths, in	verse				
Objectives	Concrete				Pictor	rial			Abstract
Y6—add several	As Year 5: Revisit and consolidate	As	Year 5:	Revisit a	ind cons	solidate	e under	standing.	As Year 5: Revisit and consolidate
numbers of increasing	understanding.								understanding.
complexity	Start with the digit of least value because if								104328 + 61731 = 166059
Including adding	regrouping happens it will affect the digits of	s	ands	spug	spu	eds	2	8	104,020 + 01,701 - 100,000
money, measure and	greater value.	Millio	Inndi	Ter	onsa	undr	Ter	Ö	
decimals with different	Combine the counters in each column and		+ŧ	÷	Ę	Ĩ			81059
numbers of decimal	Self-Seatrin November 941 Con		O		900	00	00	00	3668
points.				000	C)	600	20	60	15301
	3 4 6 2 3			000	-	000	00	00	+ 20551
	+ 5541	15	2 1			0		0	+ 20,331
					?				120,5/9
				104,328 61,731					Children ab aud dhave ab streat
									Children should have abstract
	3 4 6 2 3								supported by a pictorial or concrete if
	+ 5 5 4 1 1 1 $ -$								still needed.
									Insert zeros for place holders when
									using decimals.
	regroup as needed:								

	Decimal numbers: $3 \ 4 \ 2 \ 5 \ 1 \ 5 \ 4 \ + \ 6 \ 3 \ 6 \ 2 \ 5 \ 6 \ . \ 0 \ 1 \ 2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$		$ \begin{array}{r} 23 \cdot 361 \\ 9 \cdot 080 \\ 59 \cdot 770 \\ + 1 \cdot 300 \\ 93 \cdot 511 \\ 21 2 \end{array} $			
	Subtraction					
		Subtraction – EYFS				
ELG Number: Children co	unt reliably with numbers from 1 to 20, place the	m in order and say which number is one more or one less that	an a given number. Using quantities and			
objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.						
EYFS Vocabulary. Subtraction						
	take away, less than, the difference, fewer, counting backwards					
Objectives	Concrete	Pictorial	Abstract			



	(show 3 fingers) = (make that with arms so		
	both arms straight across our bodies) 2 (show		
	2 fingers)		
		Subtraction Year 1	
		Vocabulary Subtraction-	
Take-away, less than, the	e difference, subtract, minus, fewer, decrease, co	unting backwards, equal to, leaves, distance between, how n	nany more, how many fewer/less than,
	most, least cour	nt back, how many left, how much less is	
When children giv	e an answer, encourage them to give the answe	r in a full sentence. E.g., '7 take away 3, the difference is for	ur' '17 take away 8 is equal to 9.'
Objectives	Concrete	Pictorial	Abstract
Subtract from one-digit	Use physical objects, counters, cubes etc. to	Cross out drawn objects to show what has been taken	
and two-digit numbers	show how objects can be taken away.	away.	7—4 = 3
to 20, including 0.		00 11	16—9 = 7
		18-4=	
Taking away ones		00 44	
	6-4 = 2	99 66	
		QQ + F	
		7	
		3 ?	
	4-2=2		
		7	





	Link to addition.	Use a pictorial representation of objects to show the	Move to using numbers within the
	Use part-part whole model to model the	part-part whole model.	part-part whole model.
Represent and use number bonds and related subtraction facts within 20 Part-part whole model	inverse.		5 10 5 12 7
		Children to represent the ten frame pictorially.	14 – 4=
	Using a 10 frame to subtract - The children		How many do we take off to reach the
	may subitise how many to subtract to get to		next 10?
Subtracting to make 10	the next 10		How many do we have left to take off? 14 – 5 = 9
			You also want children to see related
			facts e.g.
			15 – 9 = 4
		Count	
		back using a number line.	





Subtraction Year 2			
Vocabulary Subtraction- equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is difference count on strategy partition tens units			
Objectives Concrete Pictorial Ab			Abstract
Subtract a two-digit number and ones, a two-digit number and tens, two two-digit numbers Partitioning to subtract without re- Grouping: 'Friendly numbers'	Use dienes to show how to partition the number when subtracting without regrouping. 34 - 13 = 21	Children draw representations of dienes and cross off. $ \begin{array}{c} $	43–21 = 22 Recording subtraction in columns supports place value and prepares for formal written methods with larger numbers. Toward the end of the year, children move to more formal recording using partitioning method: e.g., 43-21=22 40 and 3 <u>-20 and 1</u> <u>20 and 2</u>
Make ten strategy. Progression should be crossing one ten,	65 - 28 = 37	65 - 28 = 37	65 - 28 = 37



Subtraction Year 3 - This work revises and reinforces ideas from Key Stage 1, including the focus on place value			
Vocabulary Subtraction-			
equal to, take, take-away	, less, minus, subtract, leaves, distance between,	how many more, how many fewer/less than, most, least co	unt back, how many left? how much less
isdifference, count on, strategy, partition, tens units			
Objectives	Concrete	Pictorial	Abstract
To subtract numbers with up to three-digits, using formal written methods of columnar subtraction. Column subtraction (without exchanging)	Use dienes to model.	Draw representations to support understanding. $ \begin{array}{c} \hline $	Children should begin with the expanded form. Intermediate step may be needed to lead to clear subtraction understanding. 47-24=23 47-24=23 40+7 -20+3 Moving onto a more formal way as below. 507-32=25 507 -302 20+5=25 507 -223 -223 -223



Subtraction Year 4			
		Vocabulary Subtraction –	
Subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? Difference between, half, halve, how many more/fewer is than? How much			
	more/less is? Equals, sign, is	the same as, tens boundary, hundreds boundary inverse	
Objectives	Concrete	Pictorial	Abstract
Subtraction with up to 4	Children can use concrete representation of	Children can use concrete or draw a pictorial	Formal column method involving no
digits.	the columns and place value counters.	representation of the columns and place value counters.	exchanges
	ThHTO	Can physically cross out in books to solve.	3667 – 2341 =
Subtract numbers with up to 4 digits using the formal written methods appropriate of columnar subtraction	Th H T D 000 10 0 10 10 10 10 10 10 10 10 10 10 1	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	5978 – 4523 = 3667 5978 -2341 - 4523 1326 1455 Formal column method involving
where appropriate		ThHITO	6421 - 3278 =
			8442 – 2255 = Reminding children of place value when exchanging –is this a ten or a
		P	one
		3667 – 2341 = 1326	3 11 3 3 11 6 4 2 11 8 4 4 12 - 3 2 7 8 - 2 2 5 5 3 1 4 3 6 1 8 7 exchanging?
			Children should be able to represent their understanding of addition and subtraction within a bar model and a



amounts to find change.		hundredths and that they understand
£3.56 - £2.45 =£1.11		they are subtracting part of a number
		not a whole number.
H T O H T O		£2.51 - £ 1.45 = £1.06
		1
		f 7 811
		- £ 45
		F 06
		×1.00




Subtraction Year 6					
e (away), minus, decrease leave, how many are le	Vocabulary Subtraction – ft/left over? Difference between, half, halve, how many mo	re/fewer is than? How much			
Objectives Concrete Pictorial Abstract					
As with previous years, children should use	Children should use the column method when	Formal column method is used to			
place value counters to support subtraction.	subtracting tens of thousands and hundreds of	solve problems in the context of			
	thousands. As with previous years, children should use	measure, for examples, weight and			
	place value counters images and drawings to support	money. The decimal point needs to be			
Ones Tenths Hundredths Thousandths	subtraction.	lined up like all of the other place			
		value columns. Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.			
	e (away), minus, decrease leave, how many are le n =, is the same as, tens boundary, hundreds bour Concrete As with previous years, children should use place value counters to support subtraction.	Subtraction Year 6 Vocabulary Subtraction – e (away), minus, decrease leave, how many are left/left over? Difference between, half, halve, how many mo n =, is the same as, tens boundary, hundreds boundary ones boundary, tenths boundary hundredth boundary Concret Pictorial As with previous years, children should use place value counters to support subtraction. Children should use the column method when subtracting tens of thousands and hundreds of thousands. As with previous years, children should use place value counters images and drawings to support subtraction. Image: Ima			

	"1/10"5 · 34 '1 9 kg - 36 · 080 kg 69 · 339 kg
	$\frac{3}{4}$
	as 20 hundredths, therefore 0.2 is the same value as 0.20

Multiplication Multiplication – EYFS ELG Number: EYFS Vocabulary. **Multiplication Vocabulary** group, lots of, double, odd, even, equal, same Multiplication-There should be an emphasis on number exploration within EYFS. Objectives Pictorial Concrete Abstract Can solve problems Counting and other Pictures and icons that With a focus to move onto abstract What is double 4? involving doubling maths resources for encourage children to see stage. Most children will stay in the G children to make 2 equal concept of doubling as adding concrete and pictorial stage to explore two equal groups. doubling groups. 4 + 4 = 8For those children who are ready: Physical and real-life examples that encourage children to see concept of doubling as adding two equal groups. Double 1 7+7= 1+1= 2+2= 8+8= 3+3= 9+9= 4+4= 10+10= Addition 5+5= 11+11= calculations to 6+6= 12+12= model adding two equal groups. Counting in ones, twos, tens, odd and even

numbers	Finding doubles in dominoes	
Matching pairs e.g., socks, Noah's ark		
	Songs and rhymes	
	Doubles in practical contexts.	
	Groups of objects with the same number, counting how many in each group, and finding	
	how many altogether	

Multiplication Year 1						
Year 1 Multiplication Vocabulary						
	odd, even, count in twos, fives, count in tens (forwards from/backwards from), how many times?					
lots	of, groups of, once, twice, five times, ten times, m	nultiple of, times, multiply, multiply by, array, row, column, c	louble, equal groups			
Objectives	Concrete	Pictorial	Abstract			
Use of repeated	Use different objects to add equal groups	Children to draw their own visuals to support	3 x 4			
addition	Use of bead strings to understand multiplication as grouping and repeated	multiplication as repeated addition and grouping.	4 + 4 + 4 Write addition sentences to describe objects and pictures.			
	addition.	Use pictorial including number lines to solve problems. model	5 + 5 + 5=15			
	modelling 2+2+2 = 6 Use coins for repeated addition	There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3+3 15 2 3 3 3 3 3 3 3 3	2+2+2+2=10			

Count in multiples of	Count the groups as children are skip	Children make representations to show	Count in multiples of a number aloud.
twos, fives and tens,	counting, children may use their fingers as	counting in multiples.	Write sequences with multiples of
	they are skip counting.		numbers.
	the data ship data ship	0 2 4 6 8 10	2, 4, 6, 8, 10
			5, 10, 15, 20, 25, 30
		2 2 2 2 2 2 2 2 2 2 2	
		ାତି ଭାତ ପାତ ଭାତ ପାତ ଭାତ ପାତ ଭାତ ପାତ ଭାତ ପାତ	There are two apples on one plate.
		2 4 9 8 10 12 14 16 18 ZO	How many apples on 3 plates?
		Use of images, given and then created by the	
		children to support counting in 2s, 5s and 10s.	
		C C C C C C C C C C C C C C C C C C C	
	and the second second second		
		D Nases have	
		C slovers	
Understanding arrays	Use objects laid out in arrays to find the	Draw representations of arrays to show understanding	3 x 2 = 6
Children will be	answers to 2 lots 5, 3 lots of 2 etc.		2 x 5 = 10
introduced to an array			
to support			
multiplication and to			
support the			
understanding that		Arrays and repeated addition link to commutative law. 4	
multiplication is		x 3 or 3 x 4	
repeated addition	-02 -02 -02 -02		



	Children can use practical resources to solve using a bar model. $4 \times 2 = 8$	 Write the total amount. Write the total amount. 3 3 3 3 3 3 3 3 3 3 3 3 	
Children will be able to represent a multiplication calculation using an array. Write the multiplication symbol within a number sentence. Children will also understand that multiplication can be carried out in any order (commutative)	To represent arrays using a range of concrete resources.	Use representations of arrays to show different calculations and explore commutativity. Draw arrays in different ways to show the commutative () $()$ $()$ $()$ $()$ $()$ $()$ $()$	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition. $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$

		Children can draw an array as a method to solve problems.			
		Multiplication Year 3			
	- This work revis	ses and reinforces ideas from Key Stage 1.			
1	multiply, times, groups of, equal groups of, multip	le of, multiplied by, estimate, inverse, grid multiplication, ex	kpanded column		
	multiplication, pa	artition, commutative, associative, product.			
	Multiplication				
Objectives	Concrete	Pictorial	Abstract		
		Use of pictorials to support counting on in multiples	3 x 4 =12		
Count in multiples of 3, 4 and 8.	Use of practical apparatus to support counting in multiples and to reinforce understanding of times tables. 3, 4 and 8	$3 \times 4 =$ $88 88 88$ 1×10^{-1} 24 $8 \text{ groups of 3 is 24}$	4+4+4= 12 Mentally counting on in multiples. Children should use pattern spotting to support their understanding of multiples. 0, 5, 10, 15, 'Multiples of 4 end in 0,2,4,6,8. They are even numbers.'		

			it's not an even number'
Multiplying two-digit	Show the link to arrays to first introduce the	Children can draw place value counters to support their	Start with multiplying by one-digit
number by a one-digit number Grid method progressing to the formal method. Solving problems including missing number problems, integer scaling problems.	grid method.	understanding. $ \frac{744 \times 3 = 72}{400 \times 1000} $ models to the second se	numbers and showing the clear addition alongside the grid. $\frac{x 30 5}{7 210 35}$ $210 + 35 = 245$ Moving forward, multiply by a 2-digit number showing the different rows within the grid method. $10 \qquad 80$ $3 \qquad 24$ Children to add up each column to find the answer.

	x T U 4 rows of 13 Image: Constraint of the system of the sy		
		Multiplication Year 4	
		Multiplication Vocabulary	
multiply	y, multiplied by, product, short multiplication, par	tition, distributive law, commutative, groups of, multiply, tin	nes, multiples, inverse.
Objectives	Concrete	Pictorial	Abstract
		Use of pictorials to support counting on in multiples	3 x 4 =12
		3 x 4 =	4+4+4= 12
Recall multiplication	Use of practical apparatus to support counting		Mentally counting on in multiples.
facts up to 12x12.	in multiples and to reinforce understanding of		Children should use pattern spotting
	times tables.		to support their understanding of
			multiples.

	3 lots of 4	88 88 88 24 8 groups of 3 is 24	Children can help them sol struggling wit E.G. 6 x 8 I know 5 x 8 = more group c 48.	use certain ' lve the ones th: = 40 so I nee of 8 which m	'base facts' to they are d to add one heans 6 x 8 =
Multiply two-digit and three-digit numbers by a one-digit number using formal written	X 30 6 7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different	Start with mu numbers and addition	ultiplying by showing the	one-digit e clear
layout Grid method recap from year 3 for 2 digits x 1 digit Multiplying numbers by 1 digit (year 4	Use place value counters to show how we	columns to show their thinking as shown below.	× 7	30 210	5 35



		Multiplication Year 5 Multiplication Vocabulary	
composite numbers,	prime number, prime factor, cube number, square	e number, derive, factor pairs, formal written method, times	, multiply, multiplied by, multiple of,
	product, short multiplication, partition, lo	ng multiplication, scaling, decimal place, units, tenths and hu	indreds.
Objectives	Concrete	Pictorial	Abstract
Multiply numbers up to 4 digits by a one-digit number using a formal written layout. Column multiplication (short multiplication)	Formal column method with place value counters. 6 x23=	$23 \times 6 =$	Leading to multiplication 23 using a compact method <u>x.6</u> <u>138</u>

Multiply numbers up to			Children should only use the	
4 digits by two-digit			'standard' column method of long	
numbers, including long			multiplication if they can regularly get	
multiplication for two-			the	
digit numbers.			correct Long Multiplication	
			answer 43	
			using this X 65	
Column multiplication			method. 715 (5 x 43)	
(long multiplication)			+ 2580 (60 x 43)	
			2795	
Multiply and divide	Use of practical equipment to use to support	The children can use place value charts to help them see	The children can use number slides or	
whole numbers and	how this increases. This can include base ten	that the numbers are getting bigger or smaller.	can write facts such as:	
those involving decimals	and place value counters.			
by 10, 100 and 1000			45 x 100 = 4500	
		Multiplication Year 6		
		Multiplication Vocabulary		
common factors, multip	les, prime, formal written method, multiply, mult	iplied by, multiple of, product, short and long multiplication,	partition, scaling, decimal place, units,	
		tenths and hundredths.		
Objectives	Concrete	Pictorial	Abstract	
	Formal column method with place value	23 x 6 =	Leading to multiplication using a	
Consolidate Year 5 short	counters.		compact method	
multiplication.	6 x23=			
Multiply numbers up to				
4 digits by a one-digit				
number using a formal				
written layout				

Column multiplication (short multiplication)	100s 10s 1s 000 000 0000 0000 0000 000000	100s 10s 1s 0000000000000000000000000000	
Children consolidate using long multiplication for multiplying a number up to four digits by two- digit number			24 24 24 x 26= X 1 6 1 4 4 2 4 0 3 8 4
Multiplying decimals up to 2 decimal places by a single digit.			$\begin{array}{c} 6 4 1.85 \\ \times 4 \\ \hline 2 5 6 7.40 \end{array} \xrightarrow{\text{thas 2 decimal places}} \\ \hline \text{We place the decimal point so} \\ \text{that there are 2 decimal places} \end{array}$

Division								
	Division – EYFS							
		ELG Number:						
		EYFS Vocabulary.						
	Ν	lumber Division vocabulary						
hal	ve, half, share, share equally, groups share, share	equally, one each, two each, three eachgroup in pairs, thre	es tens, equal groups of,					
Objectives	Concrete	Pictorial	Abstract					
Solve problems including halving and sharing. • Halving a whole, halving a quantity of objects. • Sharing a quantity of objects.	Real life examples. Image: Children have the opportunity to physically cut objects, food or shapes in half. Counting and other maths resources for children to share into two equal groups. Image: Children to share into two equal groups.	Pictures and icons that encourage children to see concept of halving in relation to subitising, addition and subtraction knowledge. i.e. Knowing 4 is made of 2 groups of 2, so half of 4 is 2.						

	Use visual supports such as halving mats and	Bar model with pictures or icons to support		
	part-part whole, with the physical objects and	understanding of finding 2 equal parts of a number, to		
	resources that can be	further understand how two 3 😵 🗸 🙀		
	manipulated.	halves make a whole.		
		Pictures for children to		
		create and visualise 3 or		
		more equal groups.		
	Children can experience real life problems.	"We have 6 sweets. How will be share them equally so		
Understanding the	"We have 6 sweets. How will be share them	Fred and Joe have the same?"		
concept of a fair share	equally so Fred and Joe have the same?"			
	35 35			
	355 355 I	Fred Jøe		
	Allowing children to explore what is			

	fair sharing but also what is not. Allow children to make not equal groups. 35 35 35 35 35 35 35 35 35 35 35 35 35 35 35 35 35 35 35 35 36 37 38 39 30 35 36 37 38 39 30 31 32 33 34 35 36 37 38 39 30 31 35		
		Division Year 1	
	halve share share equally g	Year 1 Division Vocabulary	
Objectives	Concrete	Pictorial	Abstract
Division as sharing (sharing objects into groups)		Children use pictures or shapes to share quantities. 8 shared between 2 is 4 3 3 3 3 3 3 3 3 3 3	12 shared between 4 is 3 Share 9 buns between 3 people.
	I have 10 cubes, can you share them equally in		



		Division Year 2	
	١	Year 2 Division Vocabulary	
	groups of, equal groups of, halve, share, share	e equally, divide, divided by, divided into, repeated subtracti	on, inverse.
Objectives	Concrete	Pictorial	Abstract
Solve one-step		Children use pictures or shapes to share quantities.	12 shared between 4 is 3
problems with division		8 shared between 2 is 4	12 ÷ 4 = 3
(sharing).	SCHRONING STATE	\$\$ \$ \$ \$	Share 9 buns between 3 people.
		E E E E	9 ÷ 3 = 3
	I have 10 cubes, can you share them equally in		
	2 groups?		
	10,	12 shared between 3 is 4	
	Equal groups of amounts shared.	Children use bar modelling to show and support understanding.	
	35 35 35	12	
	35 35 35	 	

	Amounts shared into unequal groups. Marbles are left over.	12 ÷ 4 = 3	
Solve one-step problems with division as grouping.	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping $ \begin{array}{c} +3 & +3 & +3 & +3 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \hline 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ \hline 12 & 2 & 4 & = & 3 \\ \end{array} $ Using the bar model. Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group? 20 20 20 20 5 = ? 5 x ? = 20 Children to begin to use the correct mathematical symbols to show the calculation.

		20 ? 20 ÷ 5 = ?	
	- This work revises and reinforce	Division Year 3 s ideas from Key Stage 1, including the focus on place value	
quotient divisor)dividend Objectives Division with arrays	divided by, divide, divided into, groupi Concrete Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 \div 3 = 5 5 x 3 = 15 15 \div 5 = 3 3 x 5 = 15	Division Vocabulary ing, divisor, short division, remainder, inverse, quotient, divic Pictorial Draw an array and use lines to split the array into groups to make multiplication and division sentences.	AbstractFind the inverse of multiplication and division sentences by creating linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$
Division with remainders.	Divide objects between groups and see how much is left over.	$17 \div 4 =$	17 ÷ 4 = 4r1

	17 ÷ 4 = 4 r 1	????			
Divide 3-digit numbers			Divisions that divide equally with no		
by 1 digit.	96 ÷ 3	96 ÷ 3	remainder.		
Short Division		Children to use pictorial representation to	96 ÷ 3		
		support division. They could draw base ten or use an empty number line. 93 2 groups 30 groups	32 3 96 90 30 groups of 3 - 6 - 6 2 groups of 3 0		

		Division Year 4							
quatient		Division Vocabulary							
divisor)dividend	factor, divisor, divided by, o	factor, divisor, divided by, divided into, remainders, divisible by, equivalent, short division, derive,							
	dividenc	d, quotient, inverse, remainder, multiples, exchange							
Objectives	Concrete	Pictorial	Abstract						
Divide 3-digit numbers			Divisions that divide equally with no						
by 1 digit.	342 ÷ 3	342 ÷ 3	remainder.						
		Children to use pictorial representation to	342 ÷ 3						
	342 ÷ 3	support division.	114						
		Draw base ten or empty number line.	3 342						
			300 100 groups of 3						
	4 groups of 3								
	10 groups of 3	0342	-42						
	100 groups of 3								
		4 groups 10 groups 100 groups	12 4 groups of 3						
			0						
		47.4							
Division with a	Divide objects between groups and see how	1/÷4=							
remainder	much is left over.								
	1/÷4=4r1	2 4n							
		(••••• 4)	$17 \div 4 = 4r1$						
		17							
		17							







		Division Year 6							
quotient	Division Vocabulary								
divisor) dividend	divide, divided by, divid	led into, divisible by, remainder, factor, divisor, quotient, inv	erse,						
decimal place, units, tenths, hundredths, scaling, formal written methods.									
Objectives	Concrete	Abstract							
Divide numbers up to 4	Children to be secure with short division prior								
digits by a two-digit	to progression to long division.		$\begin{array}{c ccccccccccccccccccccccccccccccccccc$						
whole number using the			$\begin{array}{c ccccccccccccccccccccccccccccccccccc$						
formal written method Recapping the concrete/pictorial/abstract			$\begin{array}{c ccccccccccccccccccccccccccccccccccc$						
of division.	methods from Y3-5 will help the children to								
Long division.	understand long division.								

Addition and subtraction facts

The full set of addition calculations that pupils need to be able to solve with automaticity are shown in the table below. Pupils must also be able to solve the corresponding subtraction calculations with automaticity. Pupils must be fluent in these facts by the end of year 2 and should continue with regular practice through year 3 to secure and maintain fluency. It is essential that pupils have automatic recall of these facts before they learn the formal written methods of columnar addition and subtraction.

+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7 + 8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10 + 1	10+2	10+3	10 + 4	10+5	10+6	10+7	10+8	10+9	10 + 10

Multiplication and division facts

The full set of multiplication calculations that pupils need to be able to solve by automatic recall are shown in the table below. Pupils must also have automatic recall of the corresponding division facts.

1 x 1	1 x 2	1 x 3	1 × 4	1 x 5	1×6	1 × 7	1 x 8	1 × 9	1 x 10	1 x 11	1 x 12
11	1	15	1 ~ 4	1.0	1.0	11	1.40	1.5	1 ~ 10	1 ~ 11	1 ~ 12
2 × 1	2 × 2	2×3	2 × 4	2×5	2×6	2×7	2×8	2×9	2 × 10	2 × 11	2 × 12
3 × 1	3 × 2	3×3	3 × 4	3 × 5	3×6	3 × 7	3 × 8	3×9	3 × 10	3 × 11	3 × 12
4 × 1	4 × 2	4 × 3	4 × 4	4 × 5	4 × 6	4 × 7	4 × 8	4 × 9	4 × 10	4 × 11	4 × 12
5 × 1	5×2	5×3	5×4	5×5	5×6	5×7	5 × 8	5×9	5 × 10	5 × 11	5 × 12
6 × 1	6 × 2	6×3	6 × 4	6×5	6×6	6 × 7	6 × 8	6×9	6 × 10	6 × 11	6 × 12
7 × 1	7 × 2	7 × 3	7 × 4	7×5	7×6	7 × 7	7 × 8	7×9	7 × 10	7 × 11	7 × 12
8 × 1	8 × 2	8×3	8 × 4	8×5	8×6	8 × 7	8 × 8	8×9	8 × 10	8 × 11	8 × 12
9×1	9×2	9×3	9×4	9×5	9×6	9×7	9×8	9×9	9 × 10	9 × 11	9 × 12
10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	10 × 6	10 × 7	10 × 8	10 × 9	10 × 10	10 × 11	10 × 12
11 × 1	11 × 2	11 × 3	11 × 4	11 × 5	11 × 6	11 × 7	11 × 8	11 × 9	11 × 10	11 × 11	11 × 12
12 × 1	12 × 2	12 × 3	12 × 4	12 × 5	12 × 6	12 × 7	12 × 8	12 × 9	12 × 10	12 × 11	12 × 12

Pupils must be fluent in these facts by the end of year 4, and this is assessed in the multiplication tables check. Pupils should continue with regular practice through year 5 to secure and maintain fluency.

The 36 most important facts are highlighted in the table. Fluency in these facts should be prioritised because, when coupled with an understanding of commutativity and fluency in the formal written method for multiplication, they enable pupils to multiply any pair of numbers.

Fluency overview taken from:

Mathematics guidance: key stages 1 and 2 non-statutory guidance for the national curriculum in England June 2020

https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/1017683/Maths_guidance_KS_1_and_2.pdf

Part-Whole Model



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete



Continuous



Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help whe finding the difference.

Cubes



Benefits

Cubes can be useful to support children with the additic and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Bead Strings



-00-0000000000000000--000-000000000000-



Benefits

69

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Number Lines (labelled)



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.





40

Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10/Dienes (addition)



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children. How many ones are there altogether? Can we make an exchange? (Yes or No) How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column) How many ones do we have left? (Write in ones column) Repeat for each column.

Base 10/Dienes (subtraction)

70 72





Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children

ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently. This model is efficient with up to 4-digit numbers. Place

value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)





Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.
