

# Progression in Calculations 

EYFS - Year 6
September 2023

## Introduction

This policy supports the use of the White Rose Maths scheme, Maths Mastery, Sarah Martin and Tara Loughran resources. Progression within each area of calculation is in line with the mathematics guidance: key stages 1 and 2, non-statutory guidance for the national curriculum in England June 2020. This calculation policy is used to support children to develop a deep understanding of number and calculation. This policy has been designed to teach children through the use of concrete, pictorial and abstract representations.

- Concrete representation - a pupil is first introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and is a foundation for conceptual understanding.
- Pictorial representation - a pupil has sufficiently understood the 'hands on' experiences performed and can now relate them to representations, such as a diagram or picture of the problem.
- Abstract representation-a pupil is now capable of representing problems by using mathematical notation, for example $12 \times 2=24$.
It is important that conceptual understanding, supported by the use of representation, is secure for all procedures. Reinforcement is achieved by going back and forth between these representations


## The Mastery of Maths

At the centre of the mastery approach to the teaching of mathematics is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that are taught and used in EYFS to Year 6 in line with the requirements of the 2014 Primary National Curriculum.

## Our Calculation Policy - The Research

- Teachers should have a clear understanding of how children learn maths. Staff need to know the individual children well and develop a culture that supports children's curiosity/thinking and problem solving. (EEF)
- Manipulatives need to be used purposefully and appropriately to have an impact on learning. (EEF)
- All adults in class should have a clear, strong understanding of why a particular CPA model is being taught. (EEF)
- Representations across year groups should be consistent to connect prior learning to new learning. (DFE Maths Recover


## The rights of the mathematician

1. The right to enjoy mathematics
2. The right to have interests and preferences
3. The right to make jottings, drawings and working out
4. The right to use our own methods and approaches
5. The right to use manipulatives and resources
6. The right to reason, to talk about maths and be listened to
7. The right to make mistakes
8. The right to estimate, to guess and to conjecture
9. The right to ponder and take time
10. The right to be playful

Catherine Gripton
https://blogs.nottingham.ac.uk/primaryeducationnetwork/2020/02/14/the-rights-of-the-mathematician/

## Addition

## Addition - EYFS

ELG Number: Children count reliably with numbers from 1 to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

## EYFS Vocabulary.

## Number

Zero, number, one, two, three ... to twenty and beyond, teens numbers, eleven, twelve ... twenty, none, how many ...? count, count (up) to, count on (from, to), count back (from, to), count in ones, twos, fives, tens, is the same as, more, less, odd, even, few, pattern, pair

## Addition

Key language which should be used: total, parts and wholes, plus, add, altogether, more than, equals, 'is equal to' 'is the same as' column, place value, counting forward, more, and, make, total, double, most, count on, number line, part, part, whole

| Objectives | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| - Find the total number of items in two groups by counting all of them. (including doubling.) | Use toys and general classroom resources for children to physically manipulate, <br> group/regroup. <br> Use specific math's resources such as counters, cubes, rekenrek etc. | - <br>  $\square$ $\theta_{8}$ $\square$ $\square$ <br> (a) यma気皆 Cer <br> ${ }^{4}{ }^{3}{ }^{2}{ }^{3}{ }^{3}$ <br> Two groups of pictures so children can count the total. | A focus on symbols and numbers to form a calculation. $5+2=7$ |

(

|  | arms for example 5 (show 5 fingers) + (make a + with arms) 2 (show 2 fingers) $=$ (make that with arms so both arms straight across our bodies) 7 (show 7 fingers) |  |  |
| :---: | :---: | :---: | :---: |
| Addition Year 1 |  |  |  |
| Year 1 Vocabulary <br> add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, balancing, part, part, whole, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as' column, place value, counting forwards |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Combining two parts to make a whole: partwhole model | Use cubes to add two numbers together as a group or in a bar. <br> (Some children may still need to use real objects) <br> (4) (*) (*) *) *) *) *) <br> 92 <br> Use part-part-whole model. | The Bar Model will be continued from EYFS as a method to support problem solving involving addition, continuing with the concrete representations and moving onto using pictorial representations of objects. Some children will also move onto the abstract. |  |



| Start at the bigger number and counting on | Count on from first number (Cover first number or display as numeral). <br> $2+5$ <br> Recognise the biggest number in the calculation and count on from it (using objects for smaller number if necessary) <br> Start with the larger number on the bead string and then count of to the smaller number 1 by 1 to find the answer. <br> rekenrek | Partitioning the smaller number and use the tens number to bridge calculation. $12+5=17$ <br> Start at the larger number on the number line and count on in ones or in one jump to find the answer. | $\mathbf{6 + 8}$ becomes $\mathbf{8 + 2 + 4}$ $5+\mathbf{1 7} \text { becomes } \mathbf{1 7 + 3 + 2}$ <br> Place the larger number in your head and count on the smaller number to find your answer. |
| :---: | :---: | :---: | :---: |


| Regrouping to make 10 <br> (The 'make 10' strategy) |  | Use pictures or a number line. $\mathbf{3}+9=$ <br> Regroup or partition the smaller number using the part-part-whole model to make 10. $9+5=14$ | $7+4=11$ <br> If I am at seven, how many more do I need to make 10 ? <br> How many more do I add on now? |
| :---: | :---: | :---: | :---: |



| Addition Year 2 |  |  |  |
| :---: | :---: | :---: | :---: |
| Vocabulary <br> add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Adding 3 1-digit numbers | $4+7+6=17$ <br> Put 4 and 6 together to make 10. Add on 7. <br> Following on from making 10, making 10 with 2 of the digits (if possible) then add on the third digit. <br> When adding 3 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. | Add together three groups of objects. Draw a picture to recombine the groups to make 10. $7+6+3=16$   <br> $7+6+3=16$ $\square$ <br> 10 $\square$ | $\begin{aligned} \frac{4+7+6}{10} & =10+7 \\ & =17 \end{aligned}$ <br> Combine the two numbers that make 10 and then add on the remainder. |
| Adding a 2-digit number and ones | $17+5=22$ <br> Use ten-frame to make magic ten. | Use the part-part-whole and number line to model. | $$ |



|  |  |  | $\begin{gathered} \text { multiple of } 10.25+26=51 \\ 30+21=51 \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| Addition Year 3 <br> - This work revises and reinforces ideas from Key Stage 1, including the focus on place value |  |  |  |
| Vocabulary <br> addition add, more, and make, sum, total, altogether, double, near double, half, halve, tens, hundreds, decimal, decimal point, plus, one more, two more... ten more... one hundred more, how many more to make...? How many more is... than...? How much more is...? |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Add numbers with up to 3-digits, using formal written methods of columnar addition Column addition (no regrouping) | $269+500=769$ <br> Using manipulatives (dienes, counters) children to line up hundreds, tens and ones. <br> Move to using place value counters | Children move to drawing dienes. $269+500=769$ | Add the ones first, then the tens, then the hundreds. $\begin{array}{r} 223 \\ +114 \\ \hline 337 \end{array}$ |



|  |  |  | $\begin{array}{r} 536 \\ +85 \\ \hline 621 \\ \hline 11 \end{array}$ |
| :---: | :---: | :---: | :---: |
| Addition Year 4 |  |  |  |
| Vocabulary <br> add, addition, more, plus, increase, sum, total, altogether, double, near double, near double, half, halve, tens, hundreds, decimal, decimal point, how many more to make...? |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Add numbers with up to 4 digits <br> Using formal written methods of columnar addition where appropriate add numbers with up to 4 digits (with exchange) | Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. | Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. <br> If the children are completing a word problem, draw a bar model to represent what it's asking them to do. <br> The calculation will be shown alongside the manipulative used to see the connection. | $1,378+2,148=3,526$ <br> Continue from previous work to carry hundreds as well as tens. |


|  |  |  |  |
| :---: | :---: | :---: | :---: |
| Add decimals with 2 decimal places, including money. <br> When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20 | Calculating with decimal numbers Assign different values to dienes equipment. If a Dienes 100 block has the value of 1 , then a tens rod has a value of 0.1 and a ones cube has a value of 0.01 . These can then be used to build a conceptual understanding of the relationship between these. <br> Place value counters are another useful manipulative for representing decimal numbers. $24.2 \text { + } 13.4 \text { = }$ | Draw representations using place value grid. | As the children move on, introduce decimals with the same number of decimal places and different. <br> Money can be used here. |



| Addition Year 5 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Vocabulary <br> add, addition, more, plus, increase, sum, total, altogether, score double, near double, how many more to make...? Equals, sign, is the same as, tens, hundreds, thousands, millions, ones, tenths, inverse |  |  |  |  |  |  |  |  |  |
| Objectives | Concrete | Pictorial |  |  |  |  |  |  | Abstract |
| Y5-add numbers with more than 4 digits. <br> Add decimals with 2 decimal places, including money. <br> In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places. Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective. | As year 4. Introduce/recap decimal place value counters and model exchange for addition. <br> For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value. | Ensure a varie con | 0 <br> 10 <br> ildren <br> of dec <br> xt wh | ? <br> 4,328 <br> have experien mal places. This $n$ adding mone |  |  | 边 00 00 00 00 0 <br> ecimals with ing this into easures. | 104 | $328+61,731=166,059$ $\begin{array}{r} 81,059 \\ 3,6688 \\ 15,301 \\ 20,551 \\ \hline 20,579 \\ 1,1,1 \end{array}$ |



## Addition Year 6

## Vocabulary

add, addition, more, plus, increase, sum, total, altogether, score double, near double, how many more to make...? Equals sign =, is the same as, tens, hundreds, ones, tenths,
hundredth, thousandths, inverse



## Subtraction - EYFS

ELG Number: Children count reliably with numbers from 1 to 20 , place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

## EYFS Vocabulary. Subtraction

take away, less than, the difference, fewer, counting backwards
Objectives Concrete


|  | (show 3 fingers) = (make that with arms so both arms straight across our bodies) 2 (show 2 fingers) |  |  |
| :---: | :---: | :---: | :---: |
| Subtraction Year 1 |  |  |  |
| Take-away, less than, the difference, subtract, minus, fewer, decrease, counting backwards, equal to, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is... |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Subtract from one-digit and two-digit numbers to 20 , including 0 . <br> Taking away ones | Use physical objects, counters, cubes etc. to show how objects can be taken away. | Cross out drawn objects to show what has been taken away. <br> $7-3=?$ <br> 昭 | $\begin{gathered} 7-4=3 \\ 16-9=7 \end{gathered}$ |



Find the difference \begin{tabular}{l}
Finding the difference (using cubes, rekenrek <br>
or dienes rods) Compare objects and <br>
amounts.

 

$7-6$ or find the difference between 7 and 6 <br>
Count on using a number line to find the <br>
difference.
\end{tabular}

| Represent and use number bonds and related subtraction facts within 20 <br> Part-part whole model | Link to addition. <br> Use part-part whole model to model the inverse. | Use a pictorial representation of objects to show the part-part whole model. | Move to using numbers within the part-part whole model. <br> 10 |
| :---: | :---: | :---: | :---: |
| Subtracting to make 10 | Using a 10 frame to subtract - The children may subitise how many to subtract to get to the next 10 | Children to represent the ten frame pictorially. back using a number line. | $14-4=$ <br> How many do we take off to reach the next 10 ? <br> How many do we have left to take off? $14-5=9$ <br> You also want children to see related facts e.g. $15-9=4$ |


|  | Children could do this by subtracting a 4 | tutut | Children to represent how they have solved it e.g. <br> $14-5=9$ <br> 41 <br> 14 is made up of 5,5 and 4 so I can subtract one 5 to be left with 4 and 5 <br> 5 is made up of 4 and 1 so I can subtract 4 to make 10 and then 1 to get to 9 |
| :---: | :---: | :---: | :---: |
| Missing number problems. | Children begin by using concrete objects to support counting back to find the missing number. | Children can then use the support of a number line to support counting back to find the missing number. $13-\ldots=5$ | Children who have grasped the use of the number line concept will be taught to mentally $13-3-5=5$ <br> $13-$ $\qquad$ $=5$ <br> Children can then move on mentally counting back to find the missing |



crossing more than one

## Subtraction Year 3

- This work revises and reinforces ideas from Key Stage 1, including the focus on place value


## Vocabulary Subtraction-

equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left? how much less is...difference, count on, strategy, partition, tens units

| Objectives | Concrete | Pictorial | Abstract |
| :---: | :---: | :---: | :---: |
| To subtract numbers with up to three-digits, using formal written methods of columnar subtraction. <br> Column subtraction (without exchanging) | Use dienes to model. | Draw representations to support understanding. <br> $57-32=25$ | Children should begin with the expanded form. <br> Intermediate step may be needed to lead to clear subtraction understanding. $\begin{gathered} 47-24=23 \\ -40+7 \\ -20+4 \\ \hline 20+3 \\ \hline \end{gathered}$ <br> Moving onto a more formal way as below. <br> - Arrange in a column <br> - Subtract the ones <br> - Subtract the tens combine 435 <br> - -223 |



## Subtraction Year 4

## Vocabulary Subtraction -

Subtract, subtraction, take (away), minus, decrease, leave, how many are left/left over? Difference between, half, halve, how many more/fewer is... than...? How much more/less is...? Equals, sign, is the same as, tens boundary, hundreds boundary inverse


|  |  |  | part-part whole model. <br> Children should be able to explain that they are finding a part when they subtract, and they are finding a whole or a total when adding. |
| :---: | :---: | :---: | :---: |
| Introduce decimal subtraction through context of money | Children to be encouraged to use counters to represent numbers and take counters away to subtract. <br> Children use coins to subtract two decimal |  | Formal column method with decimals in different contexts including money $£ 3.56-£ 2.45=£ 1.11$ <br> The decimal point needs to be lined up like all the other place value columns It is important that children recognise that they are subtracting tenths and |



## Subtraction Year 5

## Vocabulary Subtraction -

Subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? Difference between, half, halve, how many more/fewer is... than...? How much more/less is...? Equals, sign, is the same as, tens boundary, hundreds boundary, thousands, millions boundary, ones boundary, tenths boundary, inverse



## Subtraction Year 6

## Vocabulary Subtraction -

Subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? Difference between, half, halve, how many more/fewer is... than...? How much more/less is...? Equals sign =, is the same as, tens boundary, hundreds boundary ones boundary, tenths boundary hundredth boundary, thousandths boundary, inverse



| Multiplication |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Multiplication - EYFS |  |  |  |  |  |
| ELG Number: |  |  |  |  |  |
| EYFS Vocabulary. <br> Multiplication Vocabulary <br> group, lots of, double, odd, even, equal, same <br> Multiplication-There should be an emphasis on number exploration within EYFS. |  |  |  |  |  |
| Objectives | Concrete | Pictorial |  | Abstract |  |
| Can solve problems involving doubling | Counting and other maths resources for children to make 2 equal groups. <br> Physical and real-life examples that encourage children to see concept of doubling as adding two equal groups. |  | Pictures and icons that encourage children to see concept of doubling as adding two equal groups. | With a focus to move onto abstract stage. Most children will stay in the concrete and pictorial stage to explore doubling For those children who are ready: |  |
| Counting in ones, twos, tens, odd and even |  |  |  |  |  |



| Multiplication Year 1 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Year 1 Multiplication Vocabulary <br> odd, even, count in twos, fives, count in tens (forwards from/backwards from), how many times? <br> lots of, groups of, once, twice, five times, ten times, multiple of, times, multiply, multiply by, array, row, column, double, equal groups |  |  |  |  |  |  |
| Objectives | Concrete |  |  |  | Children to draw their own visuals to support multiplication as repeated addition and grouping. <br> Use pictorial including number lines to solve problems. model <br> There are 3 sweets in one bag. How many sweets are in 5 bags altogether? | Abstract |
| Use of repeated addition | Use d <br> Use o multip additi <br> Repea mode Use co <br> 95. <br> 95: | 3 rent | ects | o add equal groups <br> understand ing and repeated <br> ets of objects, teacher <br> addition |  | $\begin{aligned} & 3 \times 4 \\ & 4+4+4 \end{aligned}$ <br> Write addition sentences to describe objects and pictures. <br> Grouping <br> There are 5 sweats in 1 bag. How many sweets are in 3 bags? $5+5+5=15$ |


| Count in multiples of twos, fives and tens, | Count the groups as children are skip counting, children may use their fingers as they are skip counting. | Children make representations to show counting in multiples. <br> Use of images, given and then created by the children to support counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s . | Count in multiples of a number aloud. Write sequences with multiples of numbers. $\begin{aligned} & 2,4,6,8,10 \\ & 5,10,15,20,25,30 \end{aligned}$ <br> There are two apples on one plate. How many apples on 3 plates? |
| :---: | :---: | :---: | :---: |
| Understanding arrays Children will be introduced to an array to support multiplication and to support the understanding that multiplication is repeated addition | Use objects laid out in arrays to find the answers to 2 lots 5,3 lots of 2 etc. | Draw representations of arrays to show understanding <br> Arrays and repeated addition link to commutative law. 4 | $\begin{gathered} 3 \times 2=6 \\ 2 \times 5=10 \end{gathered}$ |


| Multiplication Year 2 |  |  |  |
| :---: | :---: | :---: | :---: |
| Multiplication Vocabulary <br> odd, even, twos, fives, tens, threes, lots of, groups of, once, twice, three times, five times, ten times, multiple of, times, multiply, commutative multiply by, repeated addition, array, row, column, double. |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Count in multiples. <br> Recall and use multiplication facts for the 2,5 and 10 multiplication tables, including recognising odd and even numbers | Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. $5+5+5+5+5+5+5+5=40$ | Number lines, counting sticks and bar models should be used to show representation of counting in multiples. <br> Children make representations to show counting in <br> multiples. <br> Children can move on to using a number line to solve multiplication problems. <br> - Start at 0 <br> - Count on in the multiple | Count in multiples of a number aloud. Write sequences with multiples of numbers. $\begin{aligned} & 0,2,4,6,8,10 \\ & 0,3,6,9,12,15 \\ & 0,5,10,15,20,25,30 \end{aligned}$ <br> Then children move onto abstractly drawing their own bar model. |



|  |  | Children can draw an array as a method to solve problems. |  |
| :---: | :---: | :---: | :---: |
| - This work revises and reinforces ideas from Key Stage 1. <br> multiply, times, groups of, equal groups of, multiple of, multiplied by, estimate, inverse, grid multiplication, expanded column multiplication, partition, commutative, associative, product. |  |  |  |
| Multiplication |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Count in multiples of 3, 4 and 8. | Use of practical apparatus to support counting in multiples and to reinforce understanding of times tables. 3, 4 and 8 <br> 3 lots of 4 | Use of pictorials to support counting on in multiples $3 \times 4=$ <br> 24 <br> 8 groups of 3 is 24 | $\begin{gathered} 3 \times 4=12 \\ 4+4+4=12 \end{gathered}$ <br> Mentally counting on in multiples. Children should use pattern spotting to support their understanding of multiples. $0,5,10,15, \ldots$ <br> 'Multiples of 4 end in $0,2,4,6,8$. They are even numbers.' <br> ' 53 cannot be a multiple of 8 because |



Multiply two-digit and
three-digit numbers by
a one-digit number
using formal written
layout



| Multiply numbers up to <br> 4 digits by two-digit numbers, including long multiplication for twodigit numbers. <br> Column multiplication (long multiplication) |  |  | Children should only use the 'standard' column method of long multiplication if they can regularly get the correct answer using this method. <br> Long Multiplication $\begin{array}{r}43 \\ \times \quad 65 \\ \hline\end{array}$ $\square$ $\begin{aligned} & +\frac{2580}{2795} \\ & \underline{\underline{2795}} \end{aligned}(60 \times 43)$ |
| :---: | :---: | :---: | :---: |
| Multiply and divide whole numbers and those involving decimals by 10,100 and 1000 | Use of practical equipment to use to support how this increases. This can include base ten and place value counters. | The children can use place value charts to help them see that the numbers are getting bigger or smaller. | The children can use number slides or can write facts such as: $45 \times 100=4500$ |
| Multiplication Year 6 |  |  |  |
| Multiplication Vocabulary <br> common factors, multiples, prime, formal written method, multiply, multiplied by, multiple of, product, short and long multiplication, partition, scaling, decimal place, units, tenths and hundredths. |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Consolidate Year 5 short multiplication. | Formal column method with place value counters. $6 \times 23=$ | $23 \times 6=$ | Leading to multiplication using a compact method |
| Multiply numbers up to 4 digits by a one-digit number using a formal written layout |  |  |  |



| Division |  |  |  |
| :---: | :---: | :---: | :---: |
| Division - EYFS |  |  |  |
| ELG Number: |  |  |  |
| EYFS Vocabulary. <br> Number Division vocabulary <br> halve, half, share, share equally, groups share, share equally, one each, two each, three each...group in pairs, threes... tens, equal groups of |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Solve problems including halving and sharing. <br> - Halving a whole, halving a quantity of objects. <br> - Sharing a quantity of objects. | Real life examples. <br> Children have the opportunity to physically cut objects, food or shapes in half. <br> Counting and other maths resources for children to share into two equal groups. | Pictures and icons that encourage children to see concept of halving in relation to subitising, addition and subtraction knowledge. i.e. Knowing 4 is made of 2 groups of 2 , so half of 4 is 2 . |  |


|  | Use visual supports such as halving mats and <br> part-part whole, with the physical objects and <br> resources that can be <br> manipulated. |
| :--- | :--- |
| Bar model with pictures or icons to support |  |
| understanding of finding 2 equal parts of a number, to |  |
| further understand how two |  |
| halves make a whole. |  |
| Children can experience real life problems. |  |
| "We have 6 sweets. How will be share them |  |
| equally so Fred and Joe have the same?" |  |






|  |  | $20 \div 5=?$ |  |
| :---: | :---: | :---: | :---: |
| Division Year 3 <br> - This work revises and reinforces ideas from Key Stage 1, including the focus on place value |  |  |  |
| divisor $\frac{\text { quotient }}{\text { )dividend }}$Division Vocabularydivided by, divide, divided into, grouping, divisor, short division, remainder, inverse, quotient, dividend |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Division with arrays | Link division to multiplication by creating an array and thinking about the number sentences that can be created. $\begin{gathered} \operatorname{Eg} 15 \div 3=5 \\ 5 \times 3=15 \\ 15 \div 5=3 \\ 3 \times 5=15 \end{gathered}$ | Draw an array and use lines to split the array into groups to make multiplication and division sentences. | Find the inverse of multiplication and division sentences by creating linking number sentences. $\begin{aligned} & 7 \times 4=28 \\ & 4 \times 7=28 \\ & 28 \div 7=4 \\ & 28 \div 4=7 \\ & 28=7 \times 4 \\ & 28=4 \times 7 \\ & 4=28 \div 7 \\ & 7=28 \div 4 \end{aligned}$ |
| Division with remainders. | Divide objects between groups and see how much is left over. | $17 \div 4=$ $17$ | $17 \div 4=4 r 1$ |



| Division Year 4 |  |  |  |
| :---: | :---: | :---: | :---: |
| Division Vocabularydivisor $\frac{\text { quotient }}{\text { )dividend }}$$\quad$ factor, divisor, divided by, divided into, remainders, divisible by, equivalent, short division, derive,dividend, quotient, inverse, remainder, multiples, exchange |  |  |  |
| Objectives | Concrete | Pictorial | Abstract |
| Divide 3-digit numbers by 1 digit. | $342 \div 3$ | $342 \div 3$ <br> Children to use pictorial representation to support division. Draw base ten or empty number line. | Divisions that divide equally with no remainder. $342 \div 3$ |
| Division with a remainder | Divide objects between groups and see how much is left over. $17 \div 4=4 r 1$ | $17 \div 4=$ $17$ | $17 \div 4=4 r 1$ |





|  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Division Year 6 |  |  |  |  |  |
| Objectives | Concrete | Pictorial |  | bstract |  |
| Divide numbers up to 4 digits by a two-digit whole number using the formal written method of division. Long division. | Children to be secure with short division prior to progression to long division. <br> Recapping the concrete/pictorial/abstract methods from Y3-5 will help the children to understand long division. |  | $372 \div 15=24 \mathrm{r} 12$ |  | $1 \times 15=15$ <br> $2 \times 15=30$ <br> $3 \times 15=45$ <br> $4 \times 15=60$ <br> $5 \times 15=75$ <br> $10 \times 15=150$ |

## NUMBER FACTS FLUENCY OVERVIEW

## Addition and subtraction facts

The full set of addition calculations that pupils need to be able to solve with automaticity are shown in the table below. Pupils must also be able to solve the corresponding subtraction calculations with automaticity. Pupils must be fluent in these facts by the end of year 2 and should continue with regular practice through year 3 to secure and maintain fluency. It is essential that pupils have automatic recall of these facts before they learn the formal written methods of columnar addition and subtraction.

| $\mathbf{+}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $0+0$ | $0+1$ | $0+2$ | $0+3$ | $0+4$ | $0+5$ | $0+6$ | $0+7$ | $0+8$ | $0+9$ | $0+10$ |
| $\mathbf{1}$ | $1+0$ | $1+1$ | $1+2$ | $1+3$ | $1+4$ | $1+5$ | $1+6$ | $1+7$ | $1+8$ | $1+9$ | $1+10$ |
| $\mathbf{2}$ | $2+0$ | $2+1$ | $2+2$ | $2+3$ | $2+4$ | $2+5$ | $2+6$ | $2+7$ | $2+8$ | $2+9$ | $2+10$ |
| $\mathbf{3}$ | $3+0$ | $3+1$ | $3+2$ | $3+3$ | $3+4$ | $3+5$ | $3+6$ | $3+7$ | $3+8$ | $3+9$ | $3+10$ |
| $\mathbf{4}$ | $4+0$ | $4+1$ | $4+2$ | $4+3$ | $4+4$ | $4+5$ | $4+6$ | $4+7$ | $4+8$ | $4+9$ | $4+10$ |
| $\mathbf{5}$ | $5+0$ | $5+1$ | $5+2$ | $5+3$ | $5+4$ | $5+5$ | $5+6$ | $5+7$ | $5+8$ | $5+9$ | $5+10$ |
| $\mathbf{6}$ | $6+0$ | $6+1$ | $6+2$ | $6+3$ | $6+4$ | $6+5$ | $6+6$ | $6+7$ | $6+8$ | $6+9$ | $6+10$ |
| $\mathbf{7}$ | $7+0$ | $7+1$ | $7+2$ | $7+3$ | $7+4$ | $7+5$ | $7+6$ | $7+7$ | $7+8$ | $7+9$ | $7+10$ |
| $\mathbf{8}$ | $8+0$ | $8+1$ | $8+2$ | $8+3$ | $8+4$ | $8+5$ | $8+6$ | $8+7$ | $8+8$ | $8+9$ | $8+10$ |
| $\mathbf{9}$ | $9+0$ | $9+1$ | $9+2$ | $9+3$ | $9+4$ | $9+5$ | $9+6$ | $9+7$ | $9+8$ | $9+9$ | $9+10$ |
| $\mathbf{1 0}$ | $10+0$ | $10+1$ | $10+2$ | $10+3$ | $10+4$ | $10+5$ | $10+6$ | $10+7$ | $10+8$ | $10+9$ | $10+10$ |

## Multiplication and division facts

The full set of multiplication calculations that pupils need to be able to solve by automatic recall are shown in the table below. Pupils must also have automatic recall of the corresponding division facts.

| $1 \times 1$ | $1 \times 2$ | $1 \times 3$ | $1 \times 4$ | $1 \times 5$ | $1 \times 6$ | $1 \times 7$ | $1 \times 8$ | $1 \times 9$ | $1 \times 10$ | $1 \times 11$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2 \times 1$ | $2 \times 2$ | $2 \times 3$ | $2 \times 4$ | $2 \times 5$ | $2 \times 6$ | $2 \times 7$ | $2 \times 8$ | $2 \times 9$ | $2 \times 10$ | $2 \times 11$ |
| $2 \times 12$ |  |  |  |  |  |  |  |  |  |  |
| $3 \times 1$ | $3 \times 2$ | $3 \times 3$ | $3 \times 4$ | $3 \times 5$ | $3 \times 6$ | $3 \times 7$ | $3 \times 8$ | $3 \times 9$ | $3 \times 10$ | $3 \times 11$ |
| $3 \times 12$ |  |  |  |  |  |  |  |  |  |  |
| $4 \times 1$ | $4 \times 2$ | $4 \times 3$ | $4 \times 4$ | $4 \times 5$ | $4 \times 6$ | $4 \times 7$ | $4 \times 8$ | $4 \times 9$ | $4 \times 10$ | $4 \times 11$ |
| $4 \times 12$ |  |  |  |  |  |  |  |  |  |  |
| $5 \times 1$ | $5 \times 2$ | $5 \times 3$ | $5 \times 4$ | $5 \times 5$ | $5 \times 6$ | $5 \times 7$ | $5 \times 8$ | $5 \times 9$ | $5 \times 10$ | $5 \times 11$ |
| $6 \times 12$ |  |  |  |  |  |  |  |  |  |  |
| $6 \times 1$ | $6 \times 2$ | $6 \times 3$ | $6 \times 4$ | $6 \times 5$ | $6 \times 6$ | $6 \times 7$ | $6 \times 8$ | $6 \times 9$ | $6 \times 10$ | $6 \times 11$ |
| $7 \times 1$ | $7 \times 2$ | $7 \times 3$ | $7 \times 4$ | $7 \times 5$ | $7 \times 6$ | $7 \times 7$ | $7 \times 8$ | $7 \times 9$ | $7 \times 10$ | $7 \times 11$ |
| $7 \times 12$ |  |  |  |  |  |  |  |  |  |  |
| $8 \times 1$ | $8 \times 2$ | $8 \times 3$ | $8 \times 4$ | $8 \times 5$ | $8 \times 6$ | $8 \times 7$ | $8 \times 8$ | $8 \times 9$ | $8 \times 10$ | $8 \times 11$ |
| $9 \times 1$ | $9 \times 2$ | $9 \times 3$ | $9 \times 4$ | $9 \times 5$ | $9 \times 6$ | $9 \times 7$ | $9 \times 8$ | $9 \times 9$ | $9 \times 10$ | $9 \times 11$ |
| $9 \times 12$ |  |  |  |  |  |  |  |  |  |  |
| $10 \times 1$ | $10 \times 2$ | $10 \times 3$ | $10 \times 4$ | $10 \times 5$ | $10 \times 6$ | $10 \times 7$ | $10 \times 8$ | $10 \times 9$ | $10 \times 10$ | $10 \times 11$ |
| $11 \times 1$ | $11 \times 2$ | $11 \times 3$ | $11 \times 4$ | $11 \times 5$ | $11 \times 6$ | $11 \times 7$ | $11 \times 8$ | $11 \times 9$ | $11 \times 10$ | $11 \times 11$ |
| $11 \times 12$ |  |  |  |  |  |  |  |  |  |  |
| $12 \times 1$ | $12 \times 2$ | $12 \times 3$ | $12 \times 4$ | $12 \times 5$ | $12 \times 6$ | $12 \times 7$ | $12 \times 8$ | $12 \times 9$ | $12 \times 10$ | $12 \times 11$ |

Pupils must be fluent in these facts by the end of year 4, and this is assessed in the multiplication tables check. Pupils should continue with regular practice through year 5 to secure and maintain fluency.

The 36 most important facts are highlighted in the table. Fluency in these facts should be prioritised because, when coupled with an understanding of commutativity and fluency in the formal written method for multiplication, they enable pupils to multiply any pair of numbers.

Fluency overview taken from:
Mathematics guidance: key stages 1 and 2 non-statutory guidance for the national curriculum in England June 2020
https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment data/file/1017683/Maths guidance KS 1 and 2.pdf

## Part-Whole Model

## Bar Model (single)



## Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing par Part-whole models can be used to partition a number
into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.


## Bar Model (multiple)

Discrete

$7-3=4$

## Continuous


$7-3=4$
$2,394-1,014=1,380$

## Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure
Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use ubes and a discrete model to find the difference. This buports chidren to see how counting on can help when finding the difference.

## Cubes



## Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use tw before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they re subtracting in order to find the answer. This model of subtraction is reduction, or take away

Cubes can also be usefu to look at subtraction as difference. Here, both numbers are made and then line up to find the difference between the numbers.
Cubes are useful when working with smaller numbers but to subitise and children may miscount them.

## Bead Strings



## -90-900000000000000000--000-90000000000000000-



## Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children o investigate number bonds up to 10 .
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2+8=10$, move one bead, $3+7=10$.
Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their nowledge of number bonds to 10 and see the links to number bonds to 20 .

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

## Number Lines (labelled)



## Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)
$35+37=72$

$35+37=72$

$72-35=37$


## Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

## Base 10/Dienes (addition)


$\begin{array}{r}38 \\ +23 \\ \hline 61\end{array}$
61


## Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.
Children should first add without an exchange before moving on to addition with exchange. The representatio becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children How many ones are there altogether? Can we make an exchange? (Yes or No )
How many do we exchange? ( 10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column) How many ones do we have left? (Write in ones column) Repeat for each column.

## Base 10/Dienes (subtraction)



## Benefits

Using Base 10 or Dienes is an effective way to suppor children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.
Children should first subtract without an exchange before moving on to subtraction with exchange. When building He model, children should just make the minuend using difference to addition to avoid errors by making both numbers. Children start with the smallest place value column When there are not enough column. Whin eng
need to move to the need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract
This model is efficient with up to 4 -digit numbers. Place value counters are more efficient with larger numbers and decimals.

## Place Value Counters (addition)



## Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the mode

Children should first add without an exchange before moving on to addition with exchange. Different place al decimals. If you don't have place value counters, use r drmal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

## Glossary

Addend - A number to be added to another

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.
Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away
Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.
Total - The aggregate or the sum found by addition.

